14-2

Rework Problem 14-1 using the properties of the dressed size listed in the appendix, Table A-6(a), for a nominal 4 x 6 timber section.

Solution

Table A-6(a)

4 in x 6 in $\qquad S = 17.6 \text{ in}^3$

$$\sigma_{MAX} = \frac{M_{MAX}}{S} = \frac{24000 \text{ lb}\cdot\text{in}}{17.6 \text{ in}^3} = 1360 \text{ psi}$$

14-3

A cantilever beam has a 10 ft span (3-m) and a circular section of 4 in (100-mm) diameter. Determine the maximum flexural stress in the beam due to a 1125 lb (5-kN) concentrated load applied at the free end.

Solution.

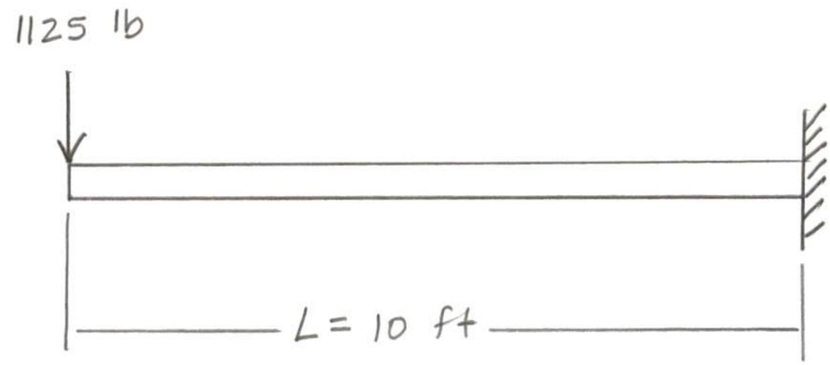
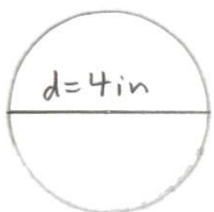


Table 13-1, case 5.

$$M_{MAX} = -Pa = -1125\text{ lb}(10\text{ ft}) = -11{,}250\text{ lb}\cdot\text{ft}$$

Circular Cross-Section



$$S = \frac{\pi d^3}{32} = \frac{\pi (4\text{in})^3}{32} = 6.3\text{ in}^3$$

$$\sigma_{MAX} = \frac{M_{MAX}}{S} = \frac{11{,}250\text{ lb-ft}\left(\frac{12\text{in}}{\text{ft}}\right)}{6.3\text{ in}^3}$$

$$= 21{,}480\text{ psi}$$ (153 MPa)

14-4

Verify the section moduli tabulated in the appendix, Tables A-1 and A-2, for the following sections:

Sx for W18 x 35

Sy for W250 x 0.71 (SI Designation) AND US Equivalent W10 x 49

Sx and Sy for S12 x 31.8

Solution.

Table A-1(a)

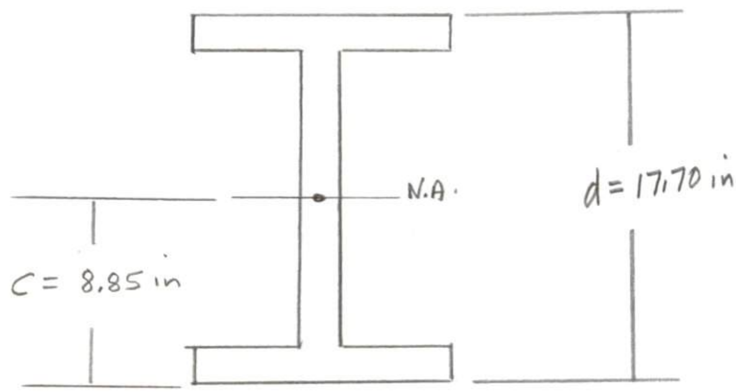
W 18 x 35

$I_x = 510 \text{ in}^4$

$S_x = 57.6 \text{ in}^3$

Check,

$$S = \frac{I}{c} = \frac{510 \text{ in}^4}{8.85 \text{ in}} = \underline{\underline{57.6 \text{ in}^3}} \checkmark$$



W 10 x 49

$I_y = 93.4 \text{ in}^4$

$S_y = 18.7 \text{ in}^3$

$b_f = 10.0 \text{ in}$

Check,

$$S_y = \frac{I_y}{c} = \frac{93.4 \text{ in}^4}{5 \text{ in}} = \underline{\underline{18.7 \text{ in}^3}} \checkmark$$

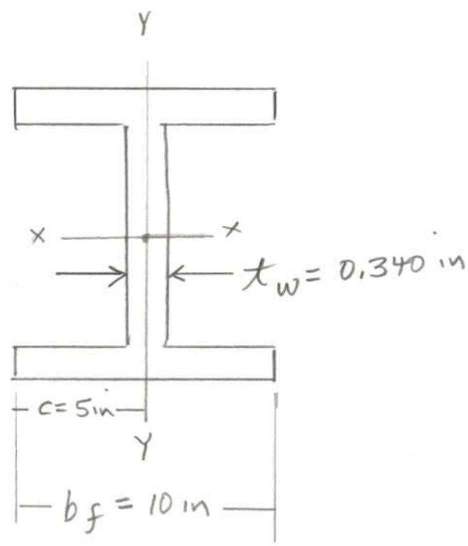


Table A-2(a)

S12 x 31.8

$d = 12.00 \text{ in}$

$b_f = 5.00 \text{ in}$

$I_x = 218 \text{ in}^4$

$S_x = 36.4 \text{ in}^3$

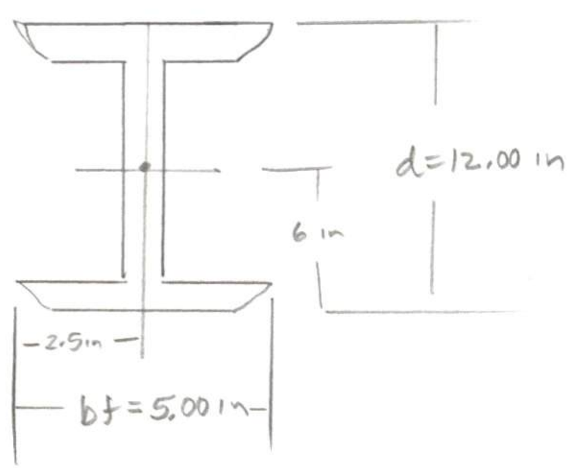
$I_y = 9.36 \text{ in}^4$

$S_y = 3.74 \text{ in}^3$

Check

$$S_x = \frac{I_x}{c} = \frac{218 \text{ in}^4}{6 \text{ in}} = \underline{\underline{36.3}} \text{ in}^3 \checkmark$$

$$S_y = \frac{I_y}{c} = \frac{9.36 \text{ in}^4}{2.5 \text{ in}} = \underline{\underline{3.74}} \text{ in}^3 \checkmark$$



14-7

A standard-weight steel pipe of 2 in (50-mm) nominal diameter is used as a post for a clothesline. The pipe is firmly embedded in a concrete base. Determine the maximum normal stress in the pipe caused by a horizontal force of 90 lb (400 N) applied at the section 6.5 ft (2 m) above the base.

Solution.

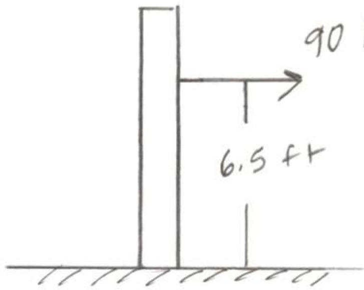


Table A-5(a)

2 in Steel Pipe $S = 0.561\ in^3$

$$M_{MAX} = 90\,lb\,(6.5\,ft) = 585\ lb \cdot ft$$

$$\sigma_{MAX} = \frac{M_{MAX}}{S} = \frac{585\ lb \cdot ft \left(\frac{12\,in}{ft}\right)}{0.561\ in^3}$$

$$= \underline{\underline{12{,}513\ psi}} \qquad (87\,MPa)$$

14-10

A timber beam has a 16 ft (5-m) simple span and a rectangular section of nominal size 6 x 16 (150 x 410). Determine the maximum flexural stress due to a concentrated load of 3600 lb (16 kN) applied at the midspan and a uniform load of 300 lb/ft (4.5 kN/m) (including the weight of the beam) over the entire length of the beam.

Solution.

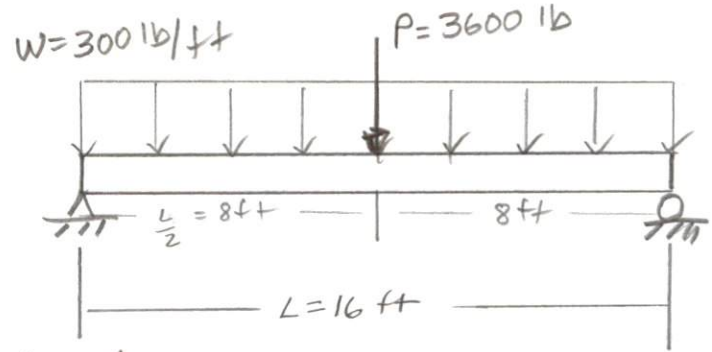


Table 13-1, case 1 and case 4

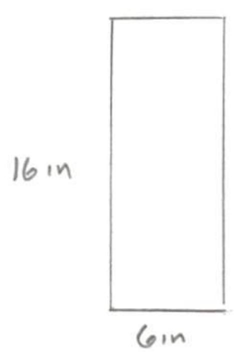
By Superposition

$$M_{MAX} = \frac{PL}{4} + \frac{wL^2}{8} = \frac{3600\,lb(16\,ft)}{4} + \frac{300\,\frac{lb}{ft}(16\,ft)^2}{8}$$

$$= 14{,}400\ lb\cdot ft + 9600\ lb\cdot ft$$

$$= 24{,}000\ lb\cdot ft$$

Nominal size (6x16)



$$\sigma_{MAX} = \frac{M_{MAX}}{S}$$

$$= \frac{24{,}000\ lb\cdot ft\left(\frac{12\,in}{ft}\right)}{220\ in^3}$$

$$= \underline{\underline{1309}}\ psi \qquad (9.45\ MPa)$$

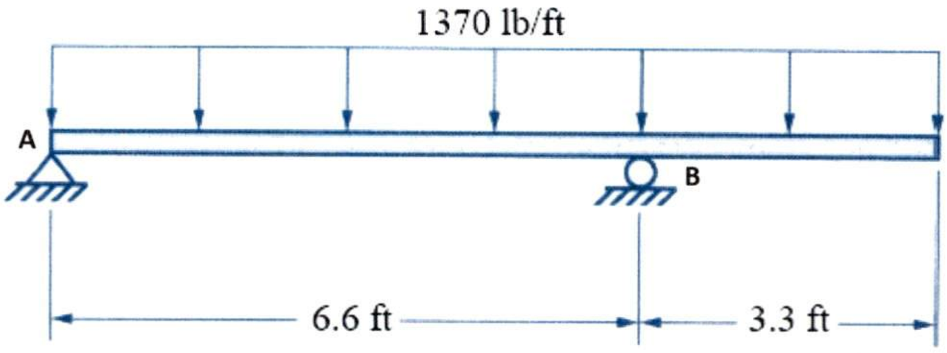
Table A-6(a)

$S = 220\ in^3$

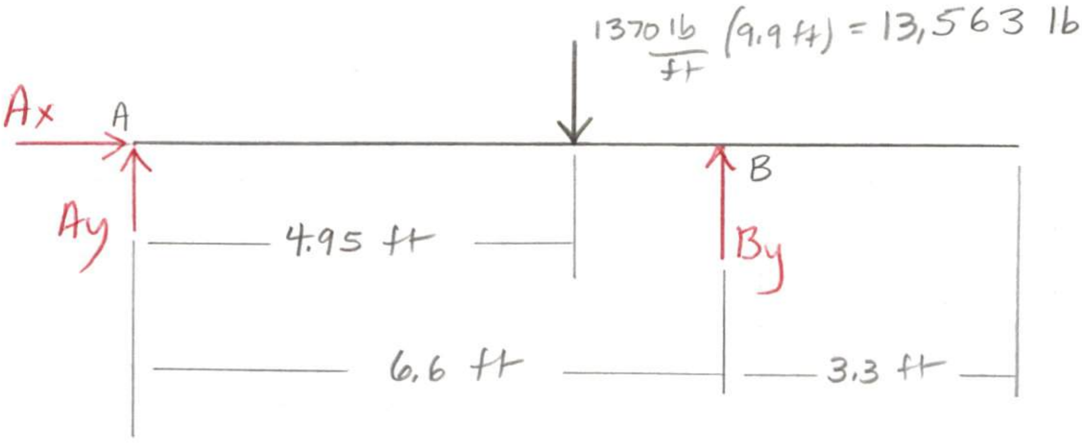
14-11

The overhanging beam shown in Fig. P14-11 has a timber section of nominal size 4 x 12 (100 x 300). Determine the maximum flexural stress due to a uniform load of 1370 lb/ft (20 kN/m) over the entire length of the beam.

Solution.



Solve for the Reactions at Supports A and B



FBD- Entire Beam

Equilibrium Equations

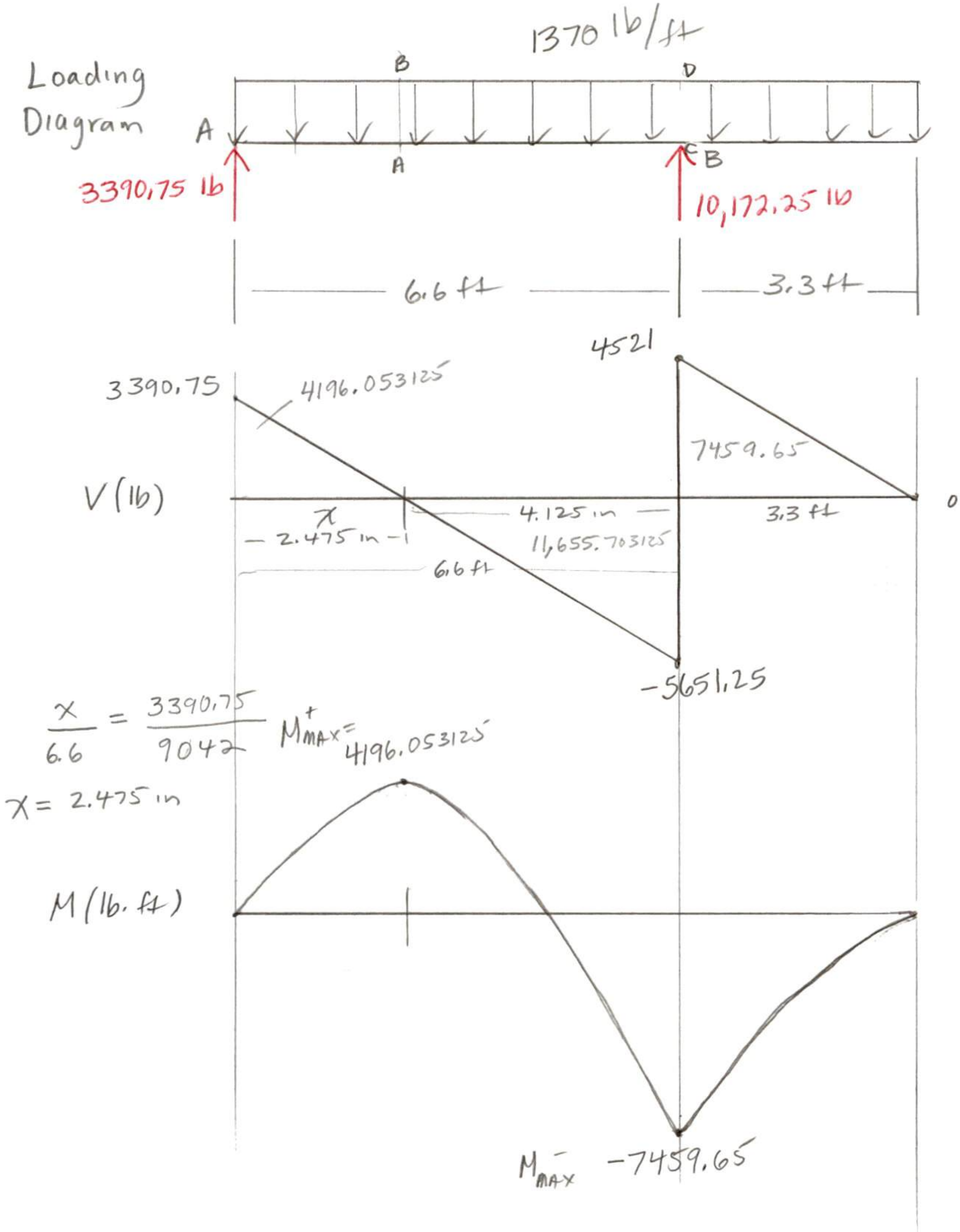
$[\Sigma F_x = 0]$ $\quad A_x = 0$

$+\circlearrowleft [\Sigma M_A = 0]$ $\quad -13{,}563 \text{ lb } (4.95 \text{ ft}) + B_y (6.6 \text{ ft}) = 0$

$$B_y = \frac{67{,}136.85 \text{ lb}\cdot\text{ft}}{6.6 \text{ ft}} = \underline{10{,}172.25 \text{ lb}} \uparrow$$

$[\Sigma F_y = 0]$ $\quad A_y - 13{,}563 \text{ lb} + B_y = 0$

$$A_y = 13{,}563 \text{ lb} - 10{,}172.25 \text{ lb} = \underline{3390.75 \text{ lb}}$$

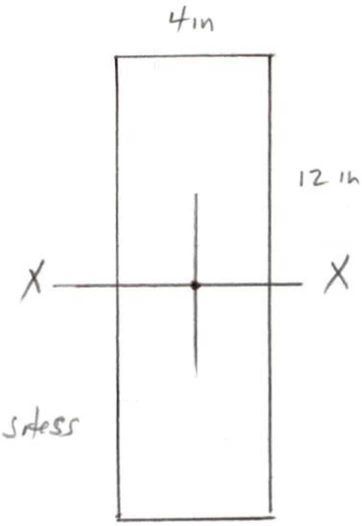


1370 lb/ft
Loading Diagram
A
B
D
A
C
B
3390.75 lb
10,172.25 lb
6.6 ft
3.3 ft
V (lb)
3390.75
4196.053125
4521
7459.65
0
x
2.475 in
4.125 in
11,655.703125
3.3 ft
6.6 ft
-5651.25
x/6.6 = 3390.75/9042
x = 2.475 in
M+MAX = 4196.053125
M (lb. ft)
M-MAX -7459.65

Table A-6(a)

Nominal size 4×12

$S = 73.8\ in^3$



For Rectangular cross-sectional area

MAX Tensile Stresss = MAX compressive stress

$$\sigma_{MAX} = \frac{|M_{MAX}|}{S}$$

$$= \frac{7459.65\ lb \cdot ft \left(\frac{12\ in}{ft}\right)}{73.8\ in^3}$$

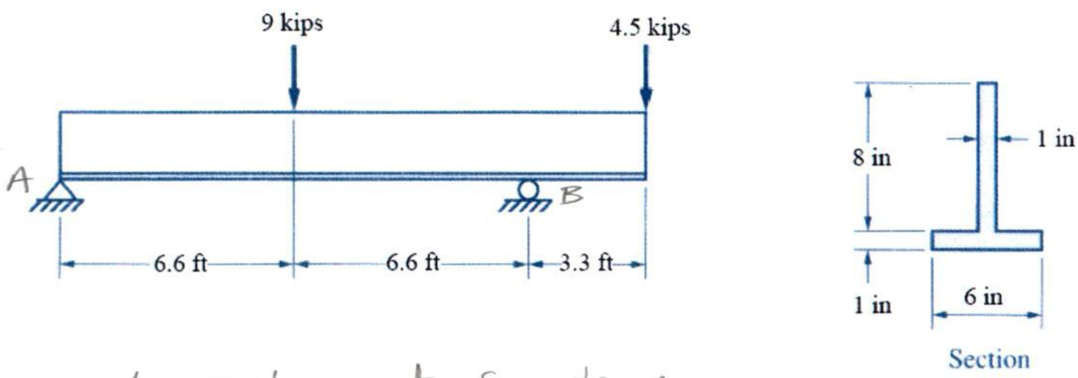
$$= \underline{\underline{1210}}\ psi$$

SI
(8.26 MPa)

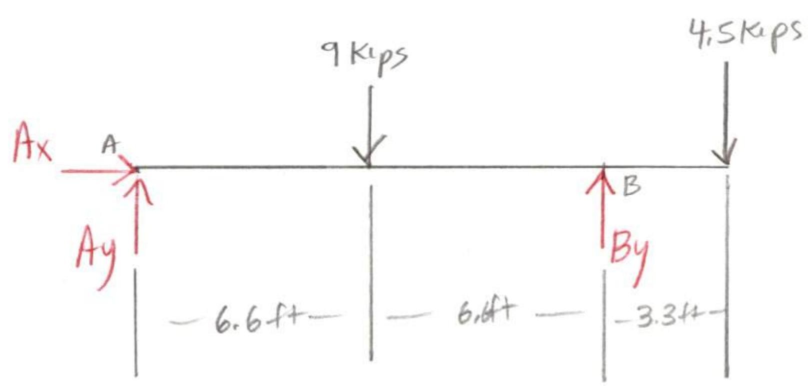
14-12

A beam with an inverted T-section is subjected to the two concentrated loads shown in Fig. P14-12. Determine the maximum tensile and compressive stresses in the beam. Neglect the weight of the beam.

Solution.



Determine the reactions at Supports A and B



FBD

Equilibrium Equations

$[\Sigma F_x = 0]$ $\qquad A_x = 0$

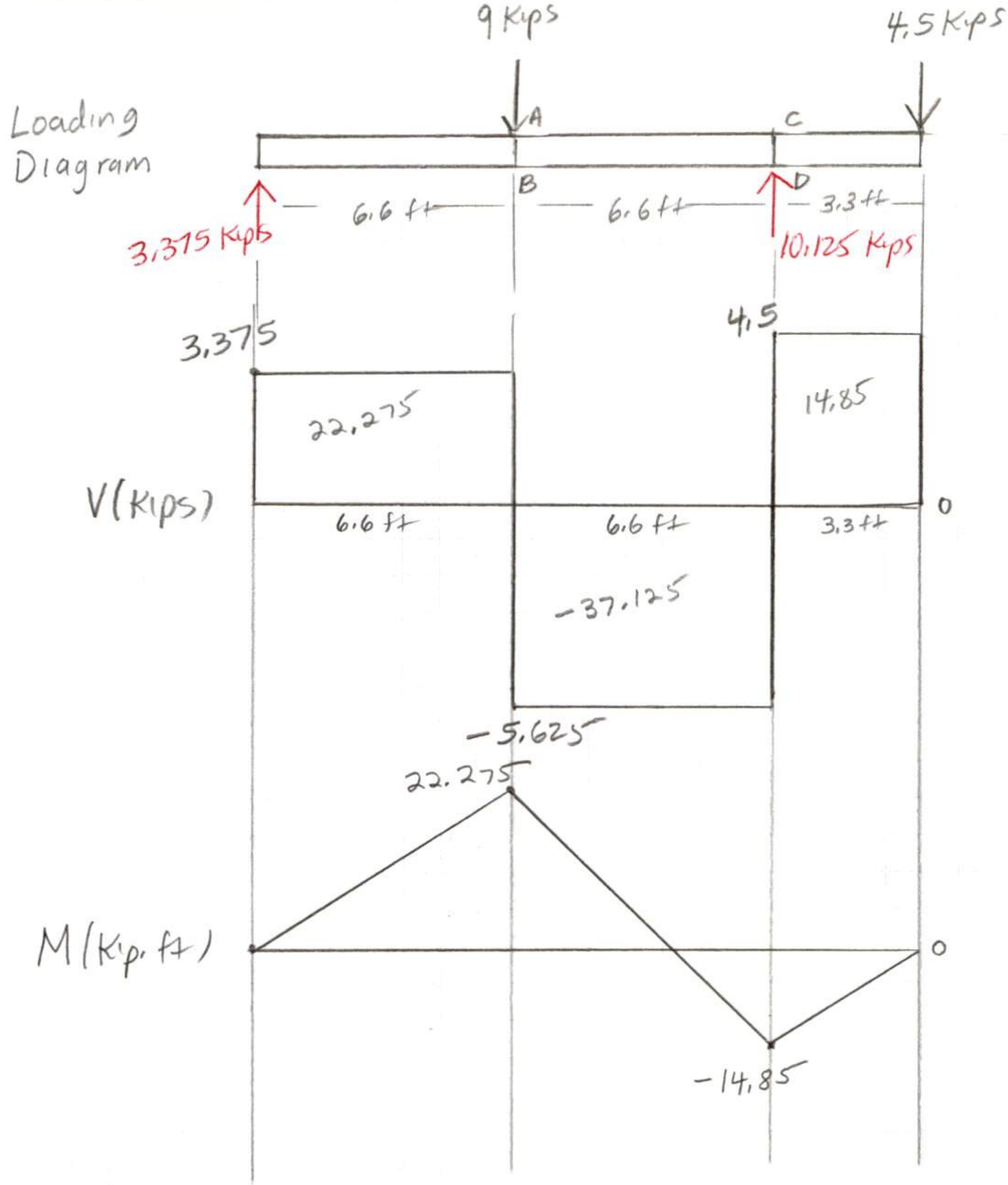
$+\circlearrowleft [\Sigma M_A = 0]$ $\qquad -9 \text{ kips } (6.6 \text{ ft}) + B_y (13.2 \text{ ft}) - 4.5 \text{ kips } (16.5 \text{ ft}) = 0$

$$B_y = \frac{133.65 \text{ kip}\cdot\text{ft}}{13.2 \text{ ft}} = \underline{\underline{10.125}} \text{ Kips} \uparrow$$

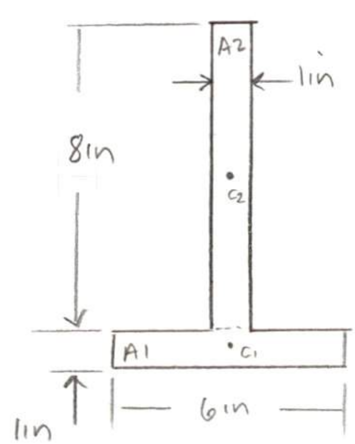
$[\Sigma F_y = 0]$ $\qquad A_y - 9 \text{ kips} + B_y - 4.5 \text{ kips} = 0$

$$A_y = 13.5 \text{ kips} - 10.125 \text{ kips} = \underline{\underline{3.375}} \text{ Kips} \uparrow$$

Find V_{MAX} and M_{MAX}



Find I



Shape	Area	y	Ay	$\bar{y}-y$	$A(\bar{y}-y)^2$	I ($bh^3/12$)
A1	6	0.5	3	2.6	40.56	0.5
A2	8	5	40	-1.9	28.88	42.67
	14		43		69.44	

$$\bar{y} = \frac{43\ in^3}{14\ in^2} = 3.1\ in$$

$$I = \Sigma\left[I + A(\bar{y}-y)^2\right]$$

$$= 43.167 + 69.44$$

$$= 112.6\ in^4$$

Maximum Tensile and Compressive Stresses

$$\sigma_A = \frac{M^{(+)}_{MAX}\, C_t}{I} = \frac{22.275 \text{ kip-ft } (5.9 \text{ in}) \left(\frac{12 \text{ in}}{\text{ft}}\right)}{112.6 \text{ in}^4} = 14 \text{ ksi (C)}$$

$$\sigma_B = \frac{M^{(+)}_{MAX}\, C_b}{I} = \frac{22.275 \text{ kip-ft } (3.1 \text{ in}) \left(\frac{12 \text{ in}}{\text{ft}}\right)}{112.6 \text{ in}^4} = 7.4 \text{ ksi (T)}$$

$$\sigma_C = \frac{M^{(-)}_{MAX}\, C_t}{I} = \frac{14.85 \text{ kip-ft } (5.9 \text{ in}) \left(\frac{12 \text{ in}}{\text{ft}}\right)}{112.6 \text{ in}^4} = 9.3 \text{ ksi (T)}$$

$$\sigma_D = \frac{M^{(-)}_{MAX}\, C_b}{I} = \frac{14.85 \text{ kip-ft } (3.1 \text{ in}) \left(\frac{12 \text{ in}}{\text{ft}}\right)}{112.6 \text{ in}^4} = 4.9 \text{ ksi (C)}$$

$$\boxed{\begin{aligned} \sigma^{(T)}_{MAX} &= 9.3 \text{ ksi} \\ \sigma^{(C)}_{MAX} &= 14 \text{ ksi} \end{aligned}}$$

14-14

Determine the allowable moment about the horizontal neutral axis that a timber beam with a nominal size 2 x 4 (50 X 100) can resist without exceeding the allowable stress of 1450 psi (10 MPa).

Solution.

$$\sigma_{allow} = 1450 \text{ psi}$$

$$M_{allow} = S\,\sigma_{allow}$$

Table A-6(a)

Nominal 2x4

$$S = 3.06 \text{ in}^3$$

$$M_{allow} = 3.06 \text{ in}^3 \left(1450 \text{ lb/in}^2\right)$$

$$= 4437 \text{ lb}\cdot\text{in} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= \underline{\underline{369.75}} \text{ lb}\cdot\text{ft}$$

14-15

Determine the allowable moment about the horizontal neutral axis that a W16 x 50 section can resist without exceeding the allowable stress of 24 ksi.

Solution.

$\sigma_{allow} = 24 \text{ ksi}$

Table A-1(a)

W16 x 50

$S = 81.0 \text{ in}^3$

$$M_{allow} = S\,\sigma_{allow}$$

$$= 24 \frac{\text{kip}}{\text{in}^2} \times 81.0 \text{ in}^3$$

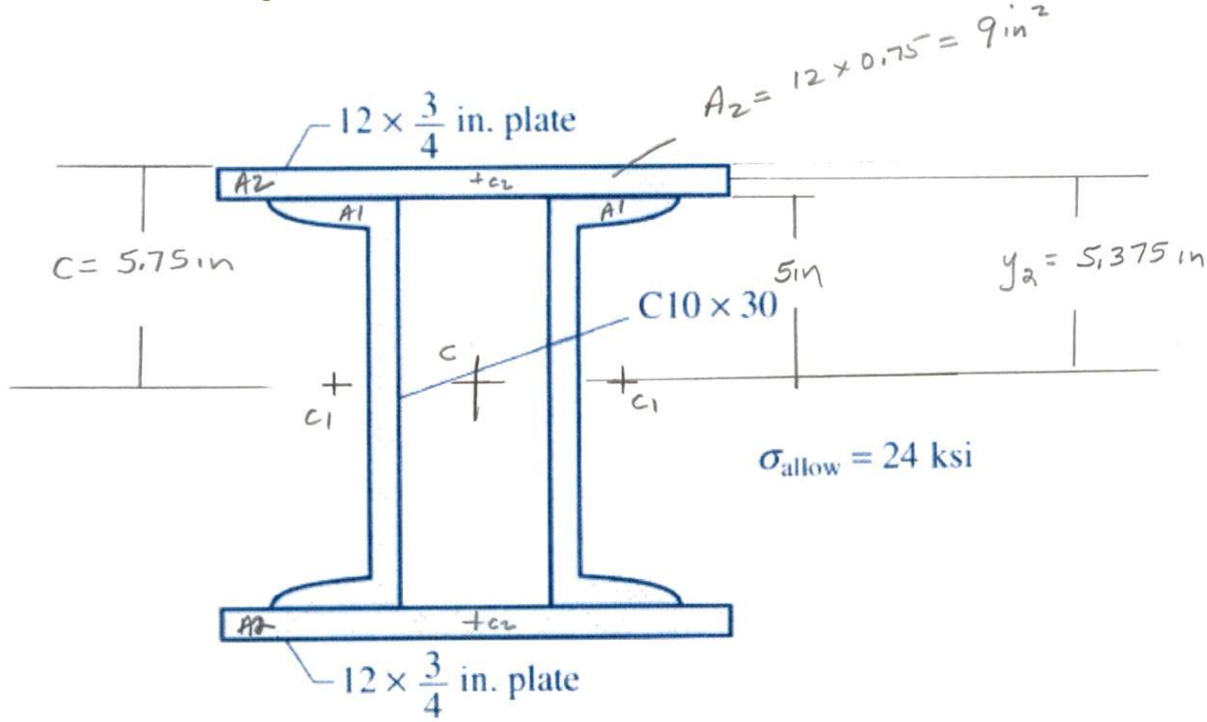
$$= 1944 \text{ kip}\cdot\text{in} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= \underline{\underline{162 \text{ kip}\cdot\text{ft}}}$$

14-16

Determine the allowable moment about the horizontal neutral axis that the beam with the built-up section shown in Fig. P14-16 can resist without exceeding the allowable stress of 24 ksi.

Solution.



$\sigma_{allow} = 24$ ksi

Deterime I_x

Table A-3(a)

C 10 × 30

$d = 10$ in

$I_{x_{(1)}} = 103 \text{ in}^4$

$$I_x = 2 I_{x_{(1)}} + 2\left[I_{x_{(2)}} + A_2 y_2^2 \right]$$

$$= 2(103 \text{ in}^4) + 2\left[\frac{12(0.75 \text{ in})^2}{12} + (12 \times 0.75)(5.375 \text{ in})^2 \right]$$

$$= 727 \text{ in}^4$$

$$S = \frac{I_x}{c} = \frac{727 \text{ in}^4}{5.75 \text{ in}} = 126.4 \text{ in}^3$$

$$M_{allow} = S\,\sigma_{allow}$$

$$= 126.4 \text{ in}^3 \,(24 \text{ kip/in}^2)$$

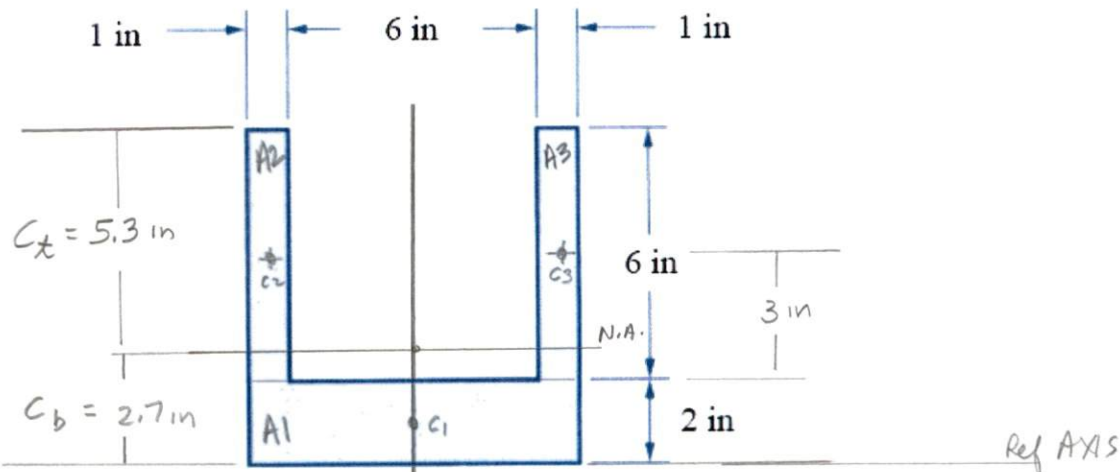
$$= 3034 \text{ kip}\cdot\text{in} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= 253 \text{ kip}\cdot\text{ft}$$

14-17

A cast-iron machine part has a channel section, as shown in Fig. P14-17. Determine the allowable positive moment about the horizontal neutral axis that the section can resist without exceeding the allowable stress of 3050 psi (21 MPa) in tension and 12,180 psi (84 MPa) in compression.

Solution.



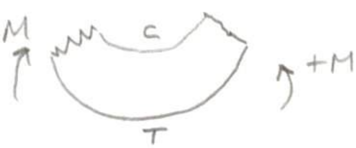
Find $\bar{y}$ and I_x

shape	A (in²)	y (in)	Ay (in³)	$(\bar{y}-y)$ (in)	$A(\bar{y}-y)^2$ (in⁴)	I (in⁴)
A1	8×2 = 16	1	16	1.7	46.24	$\frac{8(2)^3}{12} = 5.33$
A2	1×6 = 6	5	30	-2.3	31.74	$\frac{1(6)^3}{12} = 18$
A3	1×6 = 6	5	30	-2.3	31.74	18
	28		76		109.72	41.33

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{76\ in^3}{28\ in^2} = 2.7\ in$$

$$I_x = 41.33\ in^4 + 109.72\ in^4 = 151\ in^4$$

Positive Moment



Maximum Tensile Stress occurs at the bottom Fibers

$$M^{(t)}_{allow} = \frac{I_x\, \sigma^{(T)}_{allow}}{C_b} = \frac{151\ in^4\ (3050\ lb/in^2)}{2.7\ in}$$

$$= 170{,}574\ lb \cdot in \left(\frac{Kip}{1000\ \#}\right)\left(\frac{1\ ft}{12\ in}\right)$$

$$= \underline{14.2\ Kip \cdot ft}$$

Max Compressive Stress Occurs at the TOP Fibers

$$M^{(c)}_{allow} = \frac{I_x\, \sigma^{(c)}_{allow}}{C_t} = \frac{151\ in^4\ (12{,}180\ lb/in^2)}{5.3\ in}$$

$$= 347{,}015\ lb \cdot in \left(\frac{1\ ft}{12\ in}\right)\left(\frac{Kip}{1000\ lb}\right)$$

$$= \underline{29\ Kip \cdot ft}$$

ANS.

$$\boxed{M^{(+)}_{allow} = 14.2\ Kip \cdot ft}$$

14-18

Refer again to Fig. P14-17. Determine the maximum negative moment that the section can resist without exceeding the allowable stresses in tension and compression given in Problem 14-17.

Solution.

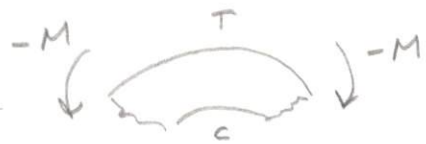
From 14-17

$I_x = 151 \text{ in}^4$

$c_b = 2.7 \text{ in}$

$c_t = 5.3 \text{ in}$

Negative Moment



MAX Tensile stress at TOP

$$M^{(-)}_{allow} = \frac{I_x \sigma^{(+)}_{allow}}{c_t} = \frac{151 \text{ in}^4 (3050 \text{ lb/in}^2)}{5.3 \text{ in}} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{\text{kip}}{1000 \text{ lb}}\right)$$

$$= 7.2 \text{ kip} \cdot \text{ft}$$

MAX Compressive Stress at Bottom

$$M^{(-)}_{allow} = \frac{I_x \sigma^{(c)}_{allow}}{c_b} = \frac{151 \text{ in}^4 (12{,}180 \text{ lb/in}^2)}{2.7 \text{ in}} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{\text{kip}}{1000 \text{ ft}}\right)$$

$$= 56.8 \text{ kip} \cdot \text{ft}$$

$$\boxed{M^{(-)}_{allow} = 7.2 \text{ kip} \cdot \text{ft}}$$

14-19

Determine the allowable load P that can be applied to the midspan of the simply supported beam shown in Fig. P14-19. The beam has a structural steel W14 x 82 section and an allowable flexural stress of 33 ksi. Neglect the weight of the beam.

Solution.

$\sigma_{allow} = 33 \text{ ksi}$

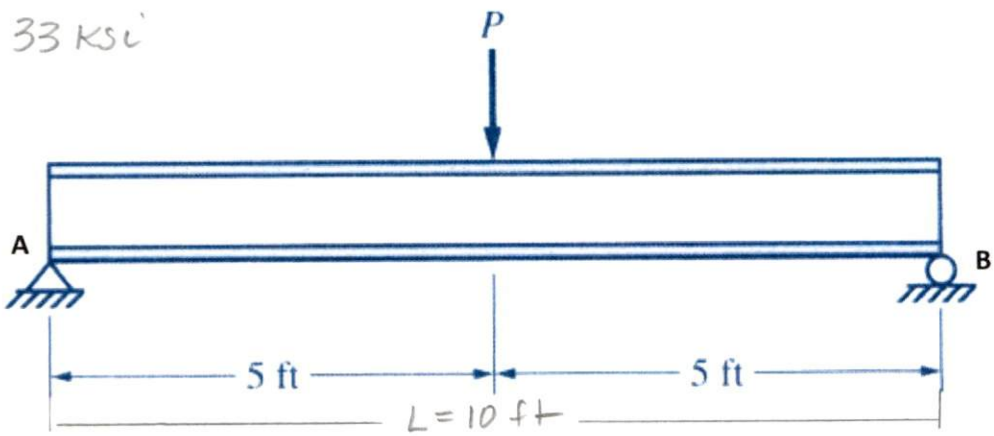


Table 13-1, case 1

$M_{MAX} = \frac{PL}{4}$

Table A-11(a)

W14x82

$S_x = 123 \text{ in}^3$

$$M_{allow} = S\,\sigma_{allow}$$
$$= 123 \text{ in}^3 \left(33 \text{ kip/in}^2\right)$$
$$= 4059 \text{ kip} \cdot \text{in}$$

$$P_{allow} = \frac{4(M_{allow})}{L} = \frac{4(4059 \text{ kip} \cdot \text{in})}{10 \text{ ft} \left(\frac{12 \text{ in}}{\text{ft}}\right)}$$

$$= \underline{\underline{135.3 \text{ kips}}}$$

14-20

See Fig. P14-20. Determine the allowable uniform load w in lb/ft that the structural steel S15 x 50 cantilever beam can carry without exceeding an allowable flexural stress of 24 ksi.

Solution.

$\sigma_{allow} = 24$ ksi

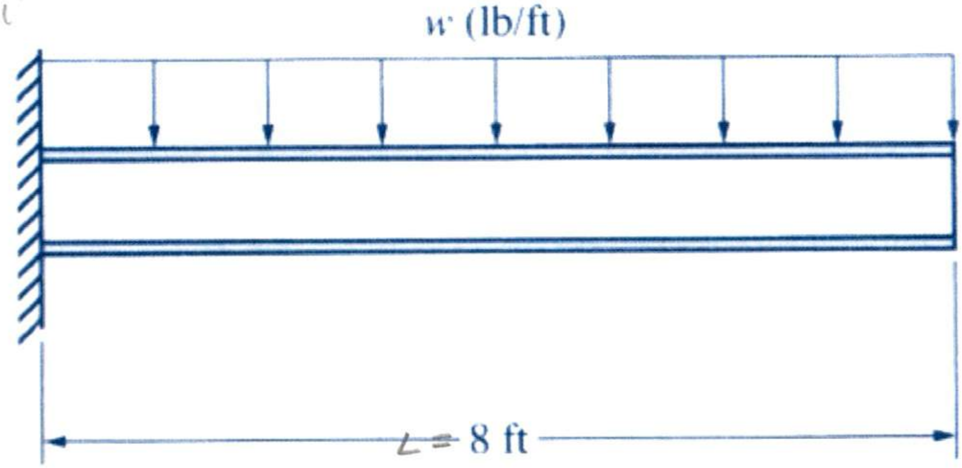


Table 13-1, Case 6

$$M_{MAX} = -\frac{wa^2}{2}$$

Table A-2(a)

S15 x 50

$S_x = 64.8 \text{ in}^3$

$$M_{allow} = S\,\sigma_{allow} = 64.8 \text{ in}^3 \left(24\,000 \frac{\text{lb}}{\text{in}^2}\right) = 1{,}555{,}200 \text{ lb}\cdot\text{in} \left(\frac{1\text{ft}}{12\text{in}}\right)$$

$$= 129{,}600 \text{ lb}\cdot\text{ft}$$

$$W = \frac{2\,M_{MAX}}{L^2} = \frac{2(129{,}600 \text{ lb}\cdot\text{ft})}{(8\text{ft})^2}$$

$$= 4050 \text{ lb/ft}$$

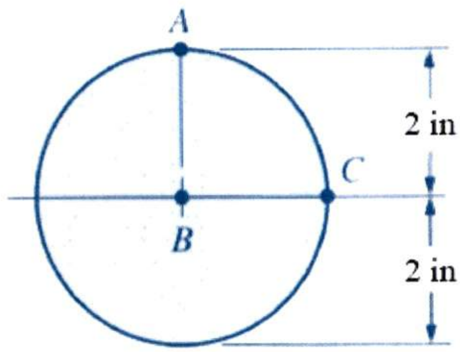
Subtracting the weight of the beam

$$W = 4050\frac{\text{lb}}{\text{ft}} - 50\frac{\text{lb}}{\text{ft}} = \underline{\underline{4000 \text{ lb/ft}}}$$

14-22

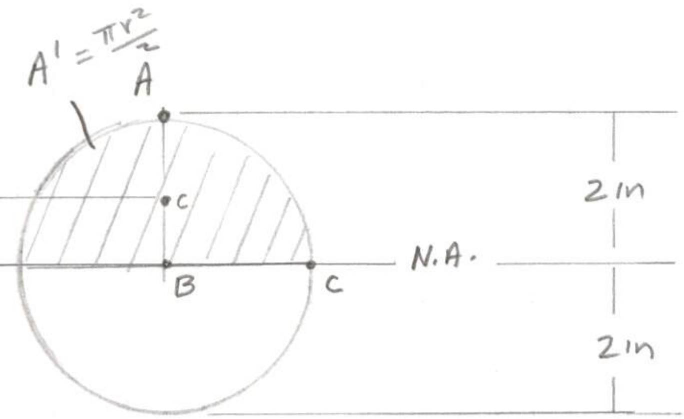
The beam of circular section in Fig. P14-22 is subjected to a maximum shear force of 3375 lb (15 kN). Determine the shear stresses at points A, B, and C.

Solution.



Shear stress @ A

$\underline{\tau_A = 0}$



$\bar{y}' = \frac{4r}{3\pi}$

$\tau_B = \tau_C = \frac{VQ}{It}$

Table 8-1

$I_x = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(2 in)^4 = 12.566370614 \ in^4$

$V = 3350 \ lb$

$$\underset{(N.A)}{\tau_B = \tau_c} = \tau_{MAX} = \frac{VQ}{It} = \frac{3350 \ lb \ \overset{A'}{\frac{\pi(2in)^2}{2}} \ \overset{\bar{y}'}{\frac{4(2in)}{3\pi}}}{12.566370614 \ in^4 \ (4 in)} = \underline{355 \ psi}$$

Also, For a circular cross-sectional Area,

$$\tau_{MAX} = \frac{4V}{3A} = \frac{4(3350 \ lb)}{3(\pi(2in)^2)} = \underline{355 \ psi} \ ✓$$

14-24

A cantilever timber beam having the full-size rectangular section shown in Fig. P14-24 is subjected to the concentrated load P at its free end. Determine the maximum allowable load P if the allowable flexural stress is 1450 psi (10 MPa) and the allowable shear stress in the beam is 116 psi (800 kPa).

Solution.

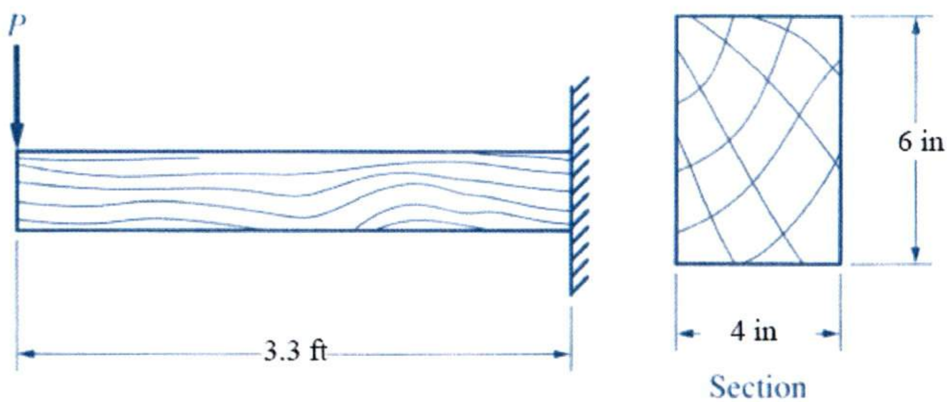


Table 13-1, case 5

$$M_{MAX} = -Pa = -PL$$

$$\sigma_{MAX} = \frac{M_{MAX}}{S} = \frac{PL}{S}$$

<u>Rectangular cross-section</u>

$$S = \frac{bh^2}{6} = \frac{4in\,(6in)^2}{6}$$

$$= 24\ in^3$$

$$P = \frac{S\,\sigma_{allow}}{L} = \frac{24\ in^3\,(1450\ lb/in^2)\left(\frac{1\ ft}{12 in}\right)}{3.3\ ft}$$

$$= 879\ lb$$

$$V_{MAX} = P$$

For rectangular Section

$$\tau_{MAX} = 1.5\,\frac{V_{MAX}}{A}$$

$$\tau_{allow} = \frac{1.5\,(P)}{A} \Rightarrow P = \frac{A\,\tau_{allow}}{1.5} = \frac{4in\,(6in)\,(116\ lb/in^2)}{1.5}$$

$$= 1856\ lb$$

$$\boxed{P_{allow} = 879\ lb}$$

14-26

Determine the maximum load P in lb that can be applied to the circular log shown in Fig. P14-26. The beam has an allowable flexural stress of 1305 psi and an allowable shear stress parallel to the grain of 123 psi.

Solution.

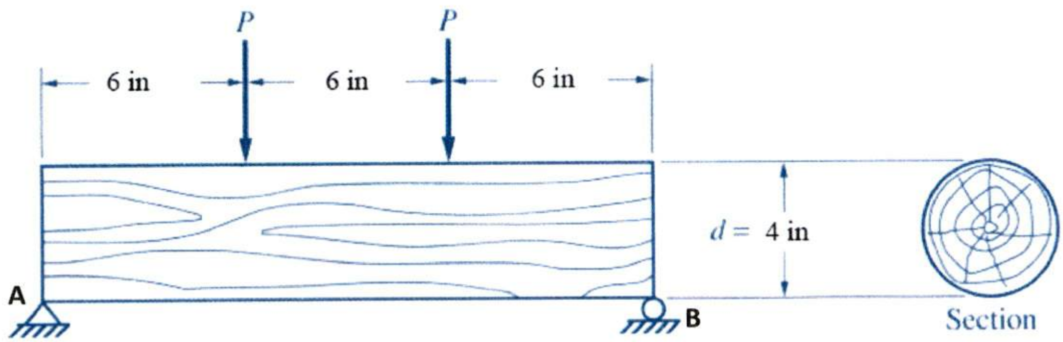


Table 13-1, case 3

$M_{MAX} = Pa$

$V_{MAX} = P$

circular cross-section

$S = \frac{\pi d^3}{32} = \frac{\pi (4in)^3}{32} = 6.28\ in^3$

Flexural stress

$\sigma_{MAX} = \frac{M_{MAX}}{S} = \frac{Pa}{S}$

$P = \frac{S\,\sigma_{allow}}{a} = \frac{6.28 in^3 (1305\ lb/in^2)}{6\ in} = \underline{\underline{1367}}\ lb$

Shear Stress

$A = \frac{\pi d^2}{4} = \frac{\pi (4in)^2}{4} = 12.56637\ in^2$

$\tau_{allow} = \tau_{MAX} = \frac{4 V_{MAX}}{3A} = \frac{4P}{3A}$

$P = \frac{3A\,\tau_{allow}}{4} = \frac{3 (12.56637 in^2)(123\ lb/in^2)}{4}$

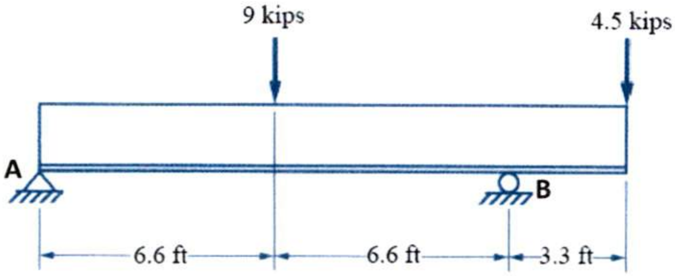
$= \underline{\underline{1160\ lb}}$

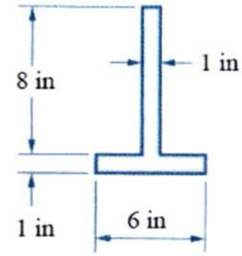
$\boxed{P_{allow} = 1160\ lb}$

14-27

Determine the maximum shear stress in the beam shown in Fig. P14-12.

Solution.



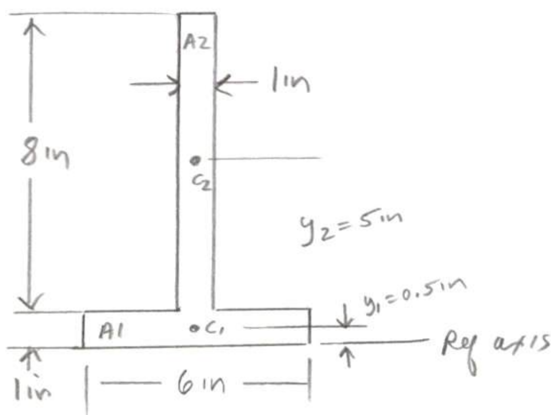


Section

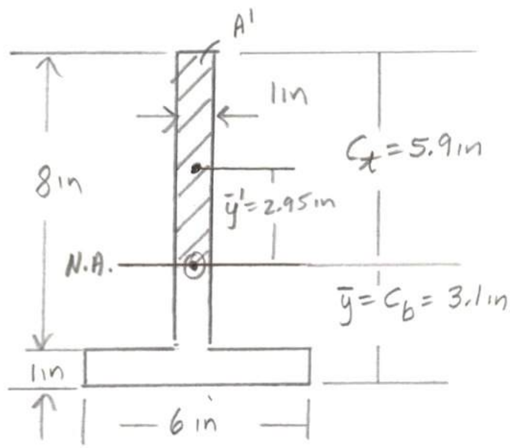
Find $\bar{y}$ (Neutral Axis)

Shape	Area (in^2)	y (in)	Ay (in^3)
A1	6	0.5	3
A2	8	5	40
	14		43

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{43\ in^3}{14\ in^2} = 3.1\ in$$



Find MAX Shear Stress



$$Q = A'\bar{y}' = (1\ in)(5.9\ in)(2.95\ in) = 17.405\ in^3$$

$$\tau_{MAX} \text{ (@ N.A)} = \frac{VQ}{It} = \frac{5.625\ Kip\ (17.405\ in^3)}{112.6\ in^4\ (1\ in)}$$

$$= 0.87\ Ksi$$

$$= \underline{870\ psi}$$

From 14-12

$I = 112.6\ in^4$

$|V_{MAX}| = 5.625\ Kip$